THE INTERDEPENDENCE ANALYSIS OF ENTREPRENEURSHIP- ECONOMIC COMPETITIVENESS LEVEL BASED ON THE LOTKA-VOLTERRA MODEL

Elena Mihaela ILIESCU[*]

Abstract

The entrepreneurship role in ensuring competitiveness and hence in in the national economies development is well known and generally accepted. The inverted influence, respectively the measure in which the competitiveness level of national economies represents a determining factor in terms of economic agents entrepreneurial behavior is however a subject less debated, especially from the empirical analysis.

Under these circumstances, the purpose of this study is to analyze using the Lotka-Volterra, based on data published between 2008-2015 in two of the most popular annual reports (The Global Entrepreneurship Monitor and the Global Competitiveness Report), the degree in which the global competitiveness index and the total entrepreneurial rate influence each other. Thus, applying the model will allow us to identify the type of interaction between the two variables (prey-predator relationship, competition and cooperation), and to estimate the value of the Total Entrepreneurial Rate and of the Global Competitiveness Index for 2016.

Keywords: *Global Competitiveness Index, Entrepreneurial Rate, Interdependency, rhythm, Lotka-Volterra.*

1. Introduction

Both the theory and the practical experience confirm the reciprocal influence between the entrepreneurship evolution and the level of global competitiveness, the latter, according to the Porter model, determinant variable of the national economic development, together with GDP/capita.

The retrospective analysis of the data published in two of the world's most well-known annual reports, The Global Entrepreneurship Monitor and The Global Competitiveness Report, regarding the evolution of the total entrepreneurial rate[1] and of the global competitiveness index[2] between 2008 and 2015, for 31 countries, doesn't reflect a linear correlation between the two variables, but a polynomial (cyclic) one.

The period under analysis is 2008-2015 since the analysis on levels of development, according to the Porter method has been introduced for the first time, within reports, in 2008. Also, the analysis was considered for analysis only 30 countries because the GEM reports do not comprise the same countries each year, and only the countries for which data are available for each year of the target range are included in the survey.

Considering all this aspects, the limited amount of data and the fact that within the entrepreneurship-economic competitiveness analysis intervenes the time variable led to the conclusion that the relationship between the two indicators cannot be analyzed using the linear regression method, which requires reconsidering the interaction of the two variables from a different perspective.

Consequently, to deepen the analysis of the interaction between the two indicators we have chosen the Lotka-Volterra model because:

- it does not require long series of data;
- allows the analysis of interdependency between two or more variables at time "t";
- describes the cyclic fluctuations of the variables pertaining to a non-linear series of data.

2. Succinct presentation of the Lotka-Volterra Model

The Lotka-Volterra model was named after the name of those that created and introduced it in literature, the American chemist and ecologist Alfred Lotka (1880-1940) and the Italian mathematician Vito Volterra (1860-1940). Thus, if originally, in 1920 the model has been created for biostatistics, to explain using the first order differential equations, the reason for the increase in the number of predatory fish, simultaneously with the reduction in the number of predatory fish in the Adriatic Sea, during the WWI, currently it is applicable in various areas, including in economy. The model was applied for the first time in the economic field in1967, when Richard Goodwin,

American mathematician and economist used the model specific equations to describe the correlation between the real wages evolution and the employment rates.

Generally, we can state that the model pursues the correlative evolution of the two indicators X and Y, interdependent indicators, the influence between the two being a special one, prey-predator type.

[*] Lecturer PhD, Faculty of Economic Sciences, "Nicolae Titulescu" University of Bucharest (email:**mag_mihaela@yahoo.com**).

[1] In order to synthesize entrepreneurship evolution, it is analyzed from the total entrepreneurial rate perspective, computed as a sum between the early entrepreneurship and the rate of consecrated entrepreneurship.

[2] It is also expressed as a percentage, as a share of the index for each analyzed country, in the total of the maximum rank.

Thus, the Y indicators, also considered as the "predator" is conditioned by the existence of X, which means that in the absence of X, Y cannot exist, and the X indicator considered "the prey" although autonomous, with its own dynamic, is conditioned also by Y's level.

The authors of the model considered that such interdependency can be modeled using the following system of differential equations:

In this context, the constants a, b, c, d express the extent to which they participate to the change of rhythm, thus[3]:

- a, d are growth coefficients of the variables under analysis, in the absence of other influences;
- b, c are called interaction coefficients and express the effect the variance of a variable has on the increase rate of the other variable.

Thus, if the value of the growth coefficients a and d are positive, the rhythm evolution is positively correlated to the evolution of the analyzed variable, while for negative values, the evolution of the rhythm is negatively correlated to the evolution of the analyzed variable.

Also, if the conditions where the interaction coefficients are non-zero (b, c 0), the relationship between the two variables can take one of the following four forms[4]:

- If b>0 and c>0 it means a cooperation relationship, mutualism;
- If b<0 and c<0 means that it is a competition relationship;
- If b<0 and c>0 it means there is a prey-predator relationship, the Y variable being the predator, and the X variable, the prey;
- If b>0 and c<0 between the two variables there is also a prey-predator relationship but, the variable X is the predator and the variable Y is the prey.

Considering the fact that these coefficients contribute together to a variance in the rhythm of the analyzed variables, the conditions a+b=1 and c+d=1 must be met.

At the same time, extreme situations must be taken into account, where one of the factors under analysis are missing. Thus, we have $\begin{cases} \frac{dX}{dt} = X \cdot (a - b \cdot Y) \quad (1) \\ \frac{dY}{dt} = Y \cdot (c \cdot X - d) \quad (2) \end{cases}$,

$a, b, c, d > 0$:

- b = 0 , meaning the Y factor is missing , equation (1) becomes,

$$\frac{dX}{dt} = X \cdot a$$

where $X = k \cdot EXP(a \cdot t)$, which means that the factor X would increase exponentially;

- c = 0, which means that factor X is missing, equation (2) becomes,

$$\frac{dY}{dt} = -d \cdot Y$$

where $Y = h \cdot EXP(-d \cdot t)$, which means that the Y factor would become extinct.

The stationary states of the system are given by:

state S1: x = 0; y = 0

state S2: x = d / c; y = a / b

a simplified version of the model is given by the equation system:

$$\begin{cases} \frac{dX}{dt} = a - b \cdot Y \quad (1') \\ \frac{dY}{dt} = c \cdot X - d \quad (2') \end{cases}$$

3. Determining the rate of change of indicators

The following study makes the transition between the theoretical analysis to the empiric analysis, by applying the Lotka-Volterra model on the empirical data regarding the evolution of the total entrepreneurial rate and of the global competitiveness index, in order to determine the relationship between the two variables and also their changing rhythm. Determining the interaction coefficients will allow estimating the value of the total entrepreneurial rate and of the global competitiveness index for 2016.

For this purpose, will be used centralized values based on data published between 2008 and 2015 in The Global Entrepreneurship Monitor and The Global Competitiveness Report regarding the total entrepreneurial rate and the global competitiveness index in national economies, for 31 countries (based on innovation and on efficiency).

In accordance with the presented model, we shall analyze the following system of equations:

$$\begin{cases} \frac{x_{n+1} - x_n}{x_n} = a - b \cdot y_n \\ \frac{y_{n+1} - y_n}{y_n} = c \cdot x_{n\cdot} - d \end{cases}$$

with the proviso that X, the prey variable, represents the entrepreneurial rate, and the Y, the

[3] Georgescu, Raluca-Mihaela, Bifurcation in biological dynamics with methods of group theory, University of Pitesti Publishing house , 2009, pp. 42.

[4] idem, pp. 43.

predator variable, represents the global competitiveness index.

The solution for the equation system led to determining the constants a, b, c and d whose values are summarized in table no.1 and table no.2.

Table no.1 Lotka-Volterra coefficients for innovation-based countries

No.	Country	a	b	c	d	Probability Ra	Probability GCI
1.	France	0,89	0,11	0,01	0,99	98,90	73,15
2.	Belgium	0,90	0,1	0,02	0,98	28,16	0,00
3.	Finland	0,93	0,07	0,01	0,99	68,11	16,14
4.	Germany	0,91	0,09	0,01	0,99	81,89	53,72
5.	Greece	0,95	0,05	0,02	0,98	56,91	43,68
6.	Italy	0,9	0,11	0,02	0,98	51,87	0,03
7.	Holland	0,94	0,06	0,01	0,99	82,63	14,30
8.	Norway	0,94	0,06	0,01	0,99	91,16	1,31
9.	Slovenia	0,91	0,09	0,02	0,98	81,49	33,26
10.	Span	0,93	0,07	0,01	0,99	32,43	21,19
11.	Great Britain	0,93	0,07	0,01	0,99	47,35	4,15
12.	USA	0,96	0,04	0,01	0,99	64,57	0,00
13.	Japan	0,92	0,08	0,01	0,99	72,28	9,14
14.	Ireland	0,94	0,06	0,01	0,99	95,66	81,61
15.	Switzerland	0,94	0,06	0,01	0,99	54,07	29,53
16.	North Korea	0,95	0,05	0,01	0,99	12,11	0,02

Source: personal computations based on data obtained using the Lotka-Volterra model

Table no.2 Lotka-Volterra coefficients for efficiency-based countries

No.	Country	a	b	c	d	Probability Ra	Probability GCI
1	Columbia	0,97	0,03	0,02	0,98	54,08	2,04
2	Argentina	0,97	0,03	0,02	0,98	52,23	61,58
3	Brazil	0,97	0,03	0,02	0,98	50,42	23,03
4	Chile	0,97	0,03	0,02	0,98	52,15	1,90
5	Croatia	0,92	0,08	0,02	0,98	74,92	21,55
6	Hungary	0,94	0,06	0,02	0,98	2,64	0,02
7	Latvia	0,95	0,05	0,02	0,98	5,52	0,49
8	Peru	0,97	0,03	0,02	0,98	6,91	11,18
9	Uruguay	0,95	0,05	0,02	0,98	30,52	5,59
10	South Africa	0,91	0,09	0,02	0,98	8,37	0,22
11	Romania	0,93	0,07	0,02	0,98	2,72	0,00
12	China	0,97	0,03	0,01	0,99	11,22	4,13
13	Malaysia	0,92	0,08	0,01	0,99	1,74	0,00
14	Mexico	0,95	0,05	0,02	0,98	31,19	1,12
15	Russia	0,88	0,12	0,02	0,98	62,05	0,23

Source: personal computations based on data obtained using the Lotka-Volterra model

The results obtained by applying the Lotka-Volterra model confirm the idea outlined thruought the paper – the interdependency relation between entrepreneurship and competitiveness. The positive values of the two constants of interaction (b and c) for all the analyzed countries reveal the symbiosis relationship, of sustenance between the two variables, not being the case of a prey-predator relationship, which confirms the conclusions outlined in previous analyses- entrepreneurship and competitiveness are two interdependent variables. However the low values of these constants, show that the evolution of the entrepreneurial rate and of the competitiveness level is mutually supported in small measure, the growth rates having a strong independent character.

Thus, we find that the competitiveness level of the national economies influence the entrepreneurial rate but, to a very small extent- between 1% and 2%, regardless of the states' development level (efficiency or innovation-based economies).

This, once again, demonstrates the subjective and conjunctural character of the decisions in the investment process, and also that, although competitiveness must represent a desideratum of all national economies, in building a the strategies for encouraging the entrepreneurial activities must be

applied also other stimulation leverages, adapting this type of behavior – developing the entrepreneurial spirit through the educational process, publicizing and promoting successful cases, ensuring a stable and predictable economic, politic and legislative context, also customizing the tools to national specificities (history, traditions, customs, culture).

Also, applying the Lotka-Volterra model confirms the validity of the reverse analysis-indeed, as all the specialty studies emphasize, entrepreneurship leaves its mark to a higher extent on the competitiveness level of the national economies, entrepreneurship contribution to increasing the competitiveness of national economies raging between 3% and 12% (on average, entrepreneurship leaves its mark on the competitiveness level of innovation-based economies of 7% and 6% for the efficiency-based economies)

Using the a, b, c and d constants enables anticipating the value of the entrepreneurial rate, and of the global competitiveness index for 2016, within the analyzed economies, the results being summarized in table no.3 and no.4.

Moreover, as it can be seen in table no.1 and 2, the probability that the foreseen values for 2016 being reached is higher for the total entrepreneurial rate rather than for the Global Competitiveness Index.

This aspect reflects the fact that entrepreneurship evolution has a more pronounced predictable character than the evolution of the global competitiveness index.

Table no. 3. Global competitiveness index and the entrepreneurial rate for 2016 for innovation-based countries- forecast based on determining the change rhythm of variables, according to Lotka – Volterra model

No.	Country	GCI % 2016	GCI % 2015	ER% 2016	ER % 2015	Δ% ER	Δ% GCI
1	Belgium	67,44	73,43	11,22	10	12,20	-8,16
2	Finland	66,1	78,57	17,51	16,8	4,23	-15,87
3	Germany	85,09	79	10,33	9,5	8,74	7,71
4	Greece	59,78	57,43	18,88	19,8	-4,65	4,09
5	Italy	56,14	63,71	9,68	9,4	2,98	-11,88
6	Holland	77,88	78	18,3	17,1	7,02	-0,15
7	Norway	85,03	73,71	11,79	12,2	-3,36	15,36
8	Slovenia	59,55	57	9,04	10,1	-10,50	4,47
9	Spain	57,95	58,43	11,47	13,4	-14,40	-0,82
10	Great Britain	84,03	75,43	12,09	12,2	-0,90	11,40
11	USA	87,9	79,86	20,83	19,2	8,49	10,07
12	South Korea	84,28	71,29	15,62	16,3	-4,17	18,22
13	Japan*	84,37	78,14	10,01	11	-9,00	7,97
14	Ireland	72,72	73	16,7	15,9	5,03	-0,38
15	Switzerland	71,29	82,29	19, 41	18,6	4,35	-13,37
16	France*	71,15	72,57	8,47	8,2	3,29	-1,96

Source: personal computations based on the data obtained using the Lotka – Volterra model

Table no. 4 Global Competitiveness Index and the entrepreneurial rate in 2016 in efficiency-based countries-forecast based on determining the variance rhythm of variables, according to the Lotka – Volterra model

Nr.	Country	GCI % 2016	GCI % 2015	ER% 2016	ER % 2015	Δ% ER	Δ% GCI
1	Mexico	33,47	61,29	29,46	27,9	5,59	-45,39
2	Columbia	69,55	61,14	28,14	27,9	0,86	13,76
3	Argentina	55,43	54,29	26,22	27,2	-3,60	2,10
4	Brazil	39,26	58,29	39,49	39,9	-1,03	-32,65
5	Chile	54,81	65,43	34,99	34,1	2,61	-16,23
6	Croatia	51,02	49	8,47	10,5	-19,33	4,12
7	Hungary	52,04	51	13,13	14,4	-8,82	2,04
8	Latvia	50,67	63,57	27,77	23,7	17,17	-20,29
9	Peru	65,47	60,14	28,59	28,8	-0,73	8,86
10	Uruguay	56,89	49,71	12,46	16,4	-24,02	14,44
11	South Africa	52,13	62,71	13,89	12,6	10,24	-16,87
12	Romania	41,72	61,71	23,66	18,3	29,29	-32,39
13	China	100	69,86	14,65	15,9	-7,86	43,14
14	Malaysia	99,83	74,71	8,68	7,7	12,73	33,62
15	Russia*	62,99	62,43	8,51	8,6	-1,05	0,90

Source: personal computations based on the data obtained by applying the Lotka – Volterra method

Applying the Lotka-Volterra model provides thus the possibility to anticipate the entrepreneurial activities evolution, which, especially in the case of anticipating an unfavorable situation, enables specific economic-financial levers' adaptation, so that, from an entrepreneurial point of view it shall follow an ascending trend.

The main inconvenient in using this method with the forecasting purpose of the total entrepreneurial rate and of the global competitiveness index, is represented by the short term forecast period- only for n+1, considering that the consequences of stimulation measures aren't felt immediately, moreover, for the case where the reduction of the total entrepreneurial rate is associated to an increasing trend of the national economies competitiveness. This is because, in this last scenario, reduction of the total entrepreneurial rate is recorded considering an increasing competitiveness, which means that during the evolution of the entrepreneurial process operate aspects related to entrepreneurs' behavior/culture (potential, beginners or consecrated) and the behavioral changes, even after a long educational strategy clearly defined and applied, are felt on the long run.

4. Conclusions

The fact that the graphical representation of the correlation between the entrepreneurship rate and the Global Competitiveness Index on level of development of the national economies reflects a polynomial relationship, limited volume of data and the fact that when analyzing this interdependence the time variable must be taken into account, led to the conclusion that the relationship between the two indicators cannot be analyzed using the linear regression method, which led to reconsidering the interaction between the two from a different perspective.

Considering this conditions, the Lotka –Volterra model enabled determining the coefficients of interaction between the two variables and estimating the value of the total entrepreneurial rate and of the global competitiveness index for 2016.

Thus, the level of competitiveness of national economies influence the entrepreneurial rate, but to a small extent- between 1% and 2%, regardless of the states' development level, and the entrepreneurial contribution to the national economies competitiveness growth, is between 3% and 12%.

The analysis and interpretation of empirical data confirms thus, the theoretical approaches, which present entrepreneurship as an important vector for the national economies' development but, it also complements them, revealing a less researched and disseminated aspect - competitiveness influence on economic agents' interest for the entrepreneurial activities.

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