ANALYZING FAT-TAILED DISTRIBUTIONS IN EMERGING CAPITAL MARKETS

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Abstract

The aim of this article focuses on analyzing the implications of fat-tailed distributions in emerging capital markets. An essential aspect that was highlighted by most empirical research, especially in terms of emerging capital markets, emphasizes the fact that extreme financial events can not be accurately predicted by the normal distribution. Fat-tailed distributions establish a very effective econometric tool in the analysis of rare events which are characterized by extreme values that occur with a relatively high frequency .The importance of exploring this particular issue derives from the fact that it is fundamental for optimal portfolio selection, derivatives valuation, financial hedging and risk management strategies. The implications of fat-tailed distributions for investment process are significant especially in the turbulent context of the global financial crisis.

***Keywords**: Non-normal distribution, estimation, statistical inference, asymmetric volatility, fat-tailed distributions, stable-Paretian distributions*

Introduction

The issues discussed in this article concern a theme of great interest in the financial area with profound implications for emerging capital markets modeling and forecasting. Emerging capital market behavior is characterized by a range of specific features, such as: volatility clustering, non-stationarity, extreme fluctuations, financial risks, deviations from normal distribution, leverage effect, time variation. This particular category of capital markets is a highly fertile research habitat where empirical analysis contradicts in certain circumstances the generally accepted theoretical substrate. Thus, this article can be perceived as a conceptual debate between different theoretical paradigms whose accuracy depend beyond any argument on the overall context in which they apply.

The normal distribution assumption has increasingly occupied a central place in computational finance. Thereby, the general idea was that capital market logarithmic returns follow a normal distribution. Despite the fact that this assumption was empirically contradicted based on various studies, the debate is far from being completed. Obviously, due to several reasons, normal distribution is preferred at the expense of more accurate, but complicated assumptions. Likewise, the central limit theorem and independence assumption play a rather significant role in financial time series analysis. Thus, from a certain point of view, an uncomplicated statistical methodology based on rigorous mathematical properties is perceived as a significant advantage in modeling continuous random variables. However, the theory is sometimes contradicted by practice, especially in the context of turbulent events such as the current global financial crisis. Strictly to topic, stock returns exhibit extreme variations due to dramatic events whose magnitude affects a wide range of investors. Moreover, the quintessence of this article constitute a rigorous analysis regarding the implications of fat-tailed distributions in circumstances of extreme financial phenomena, such as emerging capital market crashes. Consequently, econometric analysis and its degree of accuracy have a major significance for optimal portfolio selection and risk management strategies.

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Theoretical radiography of the concept

The particular issue regarding the unequivocally framing in a certain pattern of stock returns distribution remains an unsolved dilemma. However, this controversy revolves around time units. Stylized facts based on empirical evidence encourage the idea that high frequency financial time series, such as weekly, daily or intraday return data, exhibit non-normal distributions. On the other hand, to an imaginative limit it can be assumed that financial returns follow a normal distribution. Thus, for a longer time horizon, such as monthly, quarterly, half-yearly, annual or even higher, return time series are apparently perceived to be normally distributed.

Relevant information are provided by the main statistical properties, specifically Skewness and Kurtosis. In statistical terms, skewness is a measure of asymmetry of the distribution of a data series around its means. The skewness of a symmetric distribution is zero. In conclusion, in the case of normal distribution, the skewness is null. Positive skewness suggests that the distribution has a long right tail, while negative skewness implies that the distribution has a long left tail. Kurtosis measures the peakedness or flatness of the distribution of a return financial time series. The kurtosis of a normal distribution is 3, any other value generating sharp meanings in the fragile context of emerging capital markets behavior. Thus, if the kurtosis exceeds 3, the distribution is peaked (Leptokurtic) relative to the normal. On the contrary, if the kurtosis is less than 3, the distribution is flat (Platykurtic) relative to normal.

In this regard, the following figure is highly suggestive :

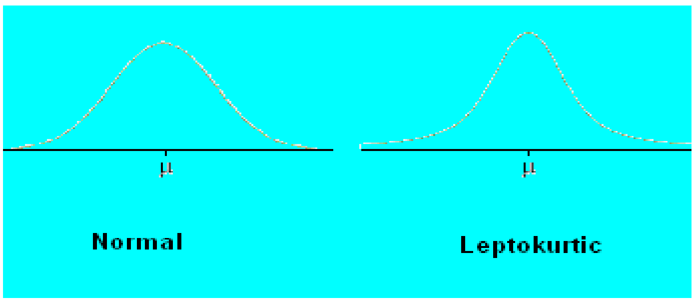

Fig. nr.1 Probability density functions for a normal and a leptokurtic distribution

Empirical studies have revealed that short term horizon stock returns are characterized by a pronounced cluster near the mean and in the tails. Likewise, fat tails involve a higher density ie a sharp attenuation of the density function in the tails. This framework is much different than a normal distribution would reflect, so this particular behavior is classified as leptokurtic distribution. In other words, this typology implies that the distribution is more peaked in the center and fat-tailed. Another interesting aspect which contradicts the classical financial theory suggests that very long term annualized returns tend to follow rather a platykurtic distribution. Making an analogy, it appears that fat–tailed distribution of returns involve the occurrence of extreme events in both directions, i.e substantial gains and major losses. In financial terms are distinguished the following categories : booms, bubbles, recessions, crisis, crashes.

In the literature this issue has been discussed with great interest in recent decades and it was emphasized by certain representative researchers (but not limited to), such as Mitchell (1915) Lévy (1925), Mandelbrot (1963), Fama (1965), Fama and Roll (1971), Rosenberg (1972), (Clark 1973), (Blattberg and Gonedes 1974), Hols et al. (1991), Tucker (1992), Mandelbrot (1997), Mandelbrot (2001), Rachev, Menn and Fabozzi (2005), Camilleri (2006), Kirchler and Huber (2007) or Mello (2008).

The analysis of emerging capital markets is based on continuously compounded return or log return, instead of using the absolute closing price variation, respectively $\Delta P = P_{t+1} - P_t$ or even on the simple net return (discrete return), which is calculated as $R_t = \frac{P_t}{P_{t-1}} - 1$.

Continuously compounded return which is calculated using the log-difference of the closing price, is calculated as follows : $r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) = \ln p_t - \ln p_{t-1}$ where p is the closing price (level) of the financial asset. Mandelbrot provides a very suggestive explanation regarding the opportunity of using continuously compounded returns in financial modeling : "The only reason for assuming continuity is that many sciences tend, knowingly or not, to copy the procedures that prove successful in Newtonian physics . . . But prices are different: mechanics involves nothing comparable." On the other hand, financial time series, such as daily stock market returns are characterized by high-frequency and excessive volatility, feature known in the literature as "volatility clustering".

The main interest in understanding stock price fluctuations derived from significant economic implications. Consequently, econometric tools and financial theoretical approach must rigorously selected.

Mitchell has empirically demonstrated that commodity prices exhibit fat-tailed distributions. In his paper "The Making and Using of Index Numbers" published in 1915, he reached an unexplored financial area and opened a new horizon for empirical research. Particular concepts discussed were in the spirit of an alternative theoretical approach, namely : time-varying volatility, interrelations (co-movements), high-peaked or fat-tailed distributions in financial asset prices.

Likewise, Mills suggested in his research papers that the normal distribution assumption is inconclusive for both natural or logarithmic distributions, but noting, however, some exceptions in particular circumstances. Significantly, it is concluded that the standard deviation derived from both natural and logarithmic numbers is being applied as a measure of dispersion. According to Mills (1927) : "a distribution may depart widely from the Gaussian type because the influence of one or two extreme price changes".

Above all, Mandelbrot (1963) is also remembered for his substantial contribution to the study of fat-tailed distributions. His initial research has relied on stable-Paretian distributions (the variance of a stable-Paretian random variable is infinite) in order to provide a justification for the fat tails empirically highlighted by other researchers. In other words, Mandelbrot based on Lévy's acceptance, suggested that tails of all non-Gaussian stable laws follow an asymptotic form of the law of Pareto, respectively : if it is considered that $C' = \sigma'^{\alpha}$ and $C'' = \sigma''^{\alpha}$ represent two constants interconnected through $\beta = (C' - C'')/(C' + C'')$ implicitly, when $u \rightarrow \infty$, $u^{\alpha}\text{Prob}(U>u) \rightarrow C' = \sigma'^{\alpha}$ and $u^{\alpha}\text{Prob}(U<-u) \rightarrow C'' = \sigma''^{\alpha}$. An essential role is played also by Cauchy law, i.e $u \rightarrow \infty$, then $u\text{Prob}(U>u) = u\text{Prob}(U<-u) \rightarrow 1/\tau$. Consequently, these two tails are Paretial if $|\beta| \neq 1$, where σ' and σ'' are the equivalent of the standard deviation of a normally distributed random variable. These particular numbers are known as the "standard positive deviation" and the "standard negative deviation." An exception is the rare case where $\beta=1$ therefore $C''=0$ (negative tail) and vice versa $\beta=-1$ therefore $C'=0$ (positive tail). Interestingly, the normal distribution was perceived just as a special case or as one of a family of stable distribution.

Mandelbrot 's approach is quite inconsistent with the central limit theorem which states that sums of random variables will converge to a normal random variable (the random variables are independent and have finite standard deviations). It is important to highlight that the central limit theorem is based on the assumption that normal distribution is the only suitable alternative distribution which is characterized by a defined standard deviation.

Regarding the concept of stable distribution's probability density function, the most relevant approaches are the following :

a) normal distribution or Gaussian : α= 2 and β is insignifiant

b) Cauchy distribution: α= 1 and β=0

c) Lévy distribution: α= 0,5 and β=1 (δ=0 and γ=1)

d) Landau distribution α= 1 and β=1

e) Holtsmark distribution α= 3/2 and β=0

In his paper "From Mandelbrot to Chaos in Economic Theory", Mirowski performs an insight regarding the log of the characteristic function for the stable distribution based on Lévy's contribution :

$$\log f(t) = \log \int_{-\infty}^{\infty} e^{iut} dP(\overline{u} < u) = i\delta t - \gamma |t|^{\alpha} \left[1 - i\beta \operatorname{sgn}(t)\omega(t,\alpha)\right]$$

where $\forall \gamma \in \Re$, $\gamma > 0$, $|\beta| \le 1$

$$\omega(t,\alpha) = \begin{cases} \tan(\alpha\pi/2) & if \quad \alpha \neq 1 \\ -(2/\pi)\log|t| & if \quad \alpha = 1 \end{cases} \quad \alpha \in [0,2]$$

$$\operatorname{sgn}(t) = \begin{cases} 1, & if \quad t > 0 \\ 0, & if \quad t = 0 \\ -1, & if \quad t < 0 \end{cases}$$

The characteristic function is based on certain real parameters, namely α, β, δ and γ. The parameter α is the Lévy index or the index of stability or characteristic exponent (on condition that $\alpha \in [0,2]$) and β is the skewness parameter (on condition that $\beta \in [-1,1]$). On the other hand γ is the scale parameter (on condition that γ>0) and $\forall \delta \in \Re$ is a location parameter.

According to Fama (1965) : There is some evidence that large changes tend to be followed by large price changes of either sign, but the dependence from this source does not seem to be too important. There is no evidence, however, that there is any dependence in the stock-price series that would be regarded as important for investment purposes. That is, the past history of the series cannot be used to increase the investor's expected profits. Regarding the fact that empirical evidence suggests the presence of an excessive number of observations near the mean and in the tails, Fama is particularly interested on some models, such as stable-Paretian distributions (mainly), mixture of normals and nonstationarity.

Certain significant conclusions were suggested by Rosenberg (1972) : "The apparent kurtosis of the empirical frequency distribution is the result of mixing distributions with predictably differing variances. . . . The results of the experiment have widespread implications for financial management and the theory of security markets. Some of these are the following:

a) the requirements for forecasts of price variance;

b) the opening of the study of the determinants of price variance as a field of economic analysis;

c) the need to respond to fluctuations in variance in portfolio management;

d) the role of fluctuations in variance, through their effect on the riskiness of investment and hence, on the appropriate risk premium, as an influence on the price level".

The implications of fat-tailed distribution for investment process

Emerging capital market behavior is characterized by certain features such as : volatility clustering, non-stationarity of price levels, leverage effect, heteroskedastic log returns, deviations from normal distribution, time variation, unpredictability, fat-tailed distributions, time-reversal asymmetry, leverage effect, chaos. In addition, the global financial crisis has emphasized these particular features. Implicitly, certain concepts such as integration, cointegration, co-movements of stock returns, interdependence of financial structures, spillover effects, market liberalization, extreme financial events have a quite sharp connotation.

Fat-tailed distributions is considered an issue of great interest in financial economics with profound implications risk management, financial assets portfolio selection, derivatives valuation and financial hedging. The implications of fat-tailed distribution for investment process are quite significant especially in the context of the global financial crisis. An emerging capital market is a fragile structure, but quite attractive for financial investors. The financial assets price behavior in emerging capital market is quite different from the classical perception of investors optimization behavior. Nevertheless, despite the fact that normal distribution represented a cornerstone in financial theory, which is used to implement fundamental financial models, such as Markowitz's modern portfolio theory, Capital Asset Pricing Model or Black-Scholes option pricing model, empirical evidence revealed the inaccuracy of this assumption.

The interest of financial investors in hedging tail risk was significantly growing, especially in the context of the global financial crisis. In addition, innovative financial tools have been developed in order to hedge tail risk and to prevent massive losses caused by extreme events. The "black swan" metaphor has acquired sharp connotations after U.S subprime crisis in mid-2007, but the consequences of ignoring them are quite dramatic. Technically speaking, tail risk is perceived as representing a higher-than-expected risk category that characterizes a financial investment that oscillates wider than three standard deviations from the mean.

In other words, a portfolio of derivatives can be liable to this risk typology if it suffer any major decrease or downward trend. Moreover, in the investment process should be taken into account that financial derivatives are designed as complex financial instruments whose main purpose is to manage the risk associated with the underlying asset (stocks, bonds, commodities, currencies, interest rates and stock market indexes). The global financial crisis that erupted in mid-2007 has generated significant losses and consequently has raised questions about the accuracy of financial econometric models based on normal distribution assumption. Nevertheless, empirical evidence on the behavior of financial asset return distributions highlighted that fat-tailed distribution is a more appropriate alternative for risk management.

Conclusions

Emerging capital market are highly volatile and difficult to forecast. The behavior of financial investors is significantly influenced by the instability of trading mechanisms in emerging capital markets. Although, developing markets represent an increasingly attractive financial environment, they are exposed to diversified risks and structural deficiencies. Consequently, financial econometric analysis and its degree of accuracy have a major significance for optimal portfolio selection and risk management strategies. Thereby, certain aspects should be considered when measuring the importance of the fat-tailed distributions concept to ensure that the consequences are significant, respectively risk management, financial assets portfolio selection, derivatives valuation and financial hedging. In other words, an inappropriate model of the underlying asset involves a mispricing and unsatisfactory hedging. On the other hand, the current global financial crisis revealed the stringent necessity of modeling and managing extreme risk in order to estimate rare events.

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