THE BONUS-MALUS SYSTEM MODELLING USING THE TRANSITION MATRIX

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Abstract

The motor insurance is an important branch of non-life insurance in many countries; in some of them, coming first in total premium income category (in Romania, for example).

The Bonus-Malus system implementation is one of the solutions chosen by the insurance companies in order to increase the efficiency in the motor insurance domain. This system has been recently introduced by the Romanian insurers as well.

In this paper I present the means for modelling the bonus-malus system using the transition matrix.

Keywords: *Bonus-Malus, system, Romanian Bonus-Malus system, transition matrix, motor insurance.*

Introduction

One of the most important problems in the Romanian insurance market is the increase in damages.

Car insurance is part of the general insurances activity and has an important share in terms of revenues, in the specific markets from Romania and from abroad. Even more, it is estimated that over 75% of the paid damages by the Romanian insurance companies are caused by the motor segment.

Dominating the portfolio of most of the composite insurance or of the general insurance companies, with increased rates which have maintained above 25%, in the last 5 years, the auto insurance can be denominated, without any doubt, a running “engine” for the Romanian insurance market. Its dominating position leads to a higher risk of market destabilization.

In Romania, the auto insurance segment has the largest market share (57%), setting the market trend and having a constant increase, against a background of:

- increase in the number of cars;
- increase in the auto park value;
- increase in the number of car accidents;
- increase of the damages rate;
- increase of the repair tariffs;
- the road infrastructure is the same.

The traffic indiscipline, the increased tariffs of the auto services, the emergence of more and more refined methods of defrauding the insurance companies and the climatic conditions that undergo a continuous change exert a great influence over the damages amount of the Motor Hull insurance segment.

Against this background, the necessity to modify the calculation system of the individual level of the bonus for the Motor Hull insurance emerged, considering the fact that the driver is the guilty party for causing an accident in most of the cases, and not the car. Thus, in setting a bonus one should consider the insured person profile: age, legal status, residence locality, driver’s license duration, destination, the frequency and manner of car usage and also the driver’s conduct in traffic. At the same time, one should not overlook the car features: the brand, the model, the original value, age, technical features (power, cylindrical capacity etc.), the additional gear and the safety systems.

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With the renewal of the auto park, the number of Motor Hull policies concluded by the insurance companies increases, as well.

The majority of the companies that subscribe to the Motor Hull policy are in search of new solutions to limit the effect of the damages dualities. Some of the solutions chosen by the insurance companies in order to cut costs on the auto segment are the deductible franchise, the implementation of the Bonus-Malus system (reward and punishment) and the use of independent systems of the costs calculation.

Definitions and remarks

Definition 1.

The Bonus-Malus system can be defined as an auto insurance system which requires the payment of a more substantial premium by the ones who are affected by damages, and a premium reduction for those who do not have undesired road events.

Malus (penalization) represents the fraction between the bonus collected by the insurer for a year of Motor Hull and the total paid or due to be paid (for damages which are about to be solved) damages by the insurer in that year. Depending on the value of that fraction, the bonus for the next year will increase up to 50%.

Bonus (reward) represents the decrease of the premium with percentages between 0- 50% and is granted for consecutive years of damage free Motor Hull.

Remark 1.

The Malus can be simple or, in many cases, a few times higher than the value of the insurance premium, depending on the insurance companies established criteria.

Another definition of the Bonus-Malus system, this time in mathematical terms, is presented below [4].

Let`s assume that, for a certain type of insurance, there is a number of rating levels J, numbered from 1 to J. In the first year of the insurance, any policy is placed in a starting level j. A policy remains in a certain rating level over a period of an insurance year. For the i class, the insurance premium will be P(i).

If a policy that is in the i rating level had during the course of the year r damages, then at the end of the year it will be transfered into the C (i,r) rating level. The C function defines transitions rules from one rating level to the other for the Bonus-Malus system. The rating levels have the following property: if k>1 then the k level is „better", having a smaller premium, so $P(k) \leq P(i)$.

So, the principal purpose of this system is that the „good" risks arrive in the high levels (of Bonus), and the „bad" risks in the lower levels (of Malus).

Definition 2.

In conclusion the Bonus-Malus system can be defined by the triplet (C,i,P), made up by the bonus elements. The (C,j) pair forms the bonus rules. The bonus elements must verify the conditions:

the P function is decreasing

the C(i,r) function is increasing in i (for a fixed r) and decreasing in r (for a fixed i).

Below are presented definitions and some basic properties of Markov chains. The purpose of these definitions is to model the Bonus-Malus system.

Definition 3.

A stochastic process $X=\{X_t, t\in\mathbf{N}\}$ in discrete time is called a (first order) Markov chain if

$$P\left(X_{t+1}=j\,|\,X_t=i,\ X_{t-1}=i_{t-1},\ \ldots,X_0=i_0\right)=P\left(X_{t+1}=j\,|\,X_t=i\right)$$

for all $j,i,i_{t-1},\ldots,i_0\in E$, where E is the state space of the Markov chain.
The probability

$$p_{ij}(t)=P\left(X_{t+1}=j\,|\,X_t=i\right) \qquad (1)$$

is called (one step) transition probability. The matrix

$$P(t)=\left(p_{ij}(t)\right) \qquad (2)$$

is called transition matrix or matrix of transition probabilities. It is a stochastic matrix, since all rows sum to one.
The probabilities

$$p_{ij}^{(r)}(t)=P\left(X_{t+r}=j\,|\,X_t=i\right) \qquad (3)$$

are called r-step transition probabilities, in particular, $p_{ij}(t)\equiv p_{ij}^{(1)}(t)$. Similarly, the matrix

$$P^{(r)}(t)=\left(p_{ij}^{(r)}(t)\right) \qquad (4)$$

is called r- step transition matrix.

Remark 2.

The probability (1) could be translate in insurance terms thus: the probability of the insured placed in the i rating level, in the t year, to pass into the j rating level, the next year.

The probability (4) could be translate in insurance terms thus: the probability of the insured that was placed in the i rating level, in the t year, to pass after r years, into the j rating level.

Definition 4.

In probability theory and statistics, the Poisson distribution is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event. Such a variable represents the number of damages during the validity of the policy , wich in non-life insurance is equal or less than one year.

If $N \sim Poisson(\lambda),\ \lambda>0$, then

$$p_x = P(N = x) = e^{-\lambda}\frac{\lambda^x}{x!}, \quad x \in \mathbf{N}, \qquad (5)$$

The Poisson distribution is often used in practice, in case of the frequency of damages, thanks both to its simplicity and to its good properties such as aditivity: if $N_i \sim Poisson(\lambda_i),\ i = 1,2$, are independent, then $N_1 + N_2 \sim Poisson(\lambda_1 + \lambda_2)$.

Definition 5.

The process $N(t)$ is a Poisson process if for some intensity $\lambda > 0$, the increments of the process have the following property:

$$N(t+r) - N(t) \sim \text{Poisson}(\lambda r) \qquad (6)$$

for all $t > 0, r > 0$ and each history $N(s),\ s \le t$.

As a result, a Poisson process has the following properties:

as increments are independent: if the intervals $(t_i, t_i + r_i),\ i = 1,2,...$, are disjoint, then the increments $N(t_i + r_i) - N(t_i)$ are independent;

the increments are stationary: $N(t+r) - N(t)$ is $\text{Poisson}(\lambda r)$ distributed for every value of *t*.

Remark 3.

Frequently a Poisson distribution is used to model the transition probabilities within a Bonus-Malus system [1]. To be more specific, the Poisson distribution describes the number of claims for an individual and the transition probabilities are determined from this claim frequency distribution. Hence, the number of claims in each year is a $\text{Poisson}(\lambda)$ variable and the probability of a year with one or more claims equals $p = 1 - e^{-\lambda}$.

Some examples of a Bonus-Malus system. The Romanian Model.

The Bonus-Malus system determines a total different conduct from the actual behavior of the Romanian insured. The percentage of the Motor Hull insured that were compensated in the course of a year, in our country is over 50%, the damages ranging between minor scratches and broken rear mirrors.

In the countries where the Bonus-Malus system has been developed, this type of damages are usually solved by the insured on his expenses, in order to maintain the Bonus-Malus scoring and not to fall, implicitly, in a superior risk category. And the increase of the rating level is much more expensive than a simple repairment.

Every country has his own Bonus-Malus system, the wheel having been reinvented quite a few times. First, a basic premium is determined using rating factors like weight, catalogue price or capacity of the car, type of use of the care (privately or for the business) and of course the type of coverage (Motor Hull Insurance, Motor TPL Insurance or a mixture). This is the premium that drivers without a known claims history have to pay. The bonus and malus for good and bad claims experience are implemented through the use of a so-called Bonus-Malus scale.

One ascends one step, getting a greater bonus, after a claim-free year, and descends one or several steps after having filed one or more claims.

There are different rating factors and a different Bonus-Malus scale for different countries [3].

The Romanian Model. The transition matrix.

The Bonus-Malus scale, including the percentages of the basic premium to be paid and the transitions made after 0,1 or more claims, is depicted in Tabel 1. In principle new insureds enter at the step with premium rate (adjustment coefficient) 100%. This is only one Bonus-Malus model used by Romanian insurance companies.

Table no. 1

Transitions rules and premium rates for one the Romanian Bonus-Malus system

Rating level (class)	Premium rate (percentage)	New Bonus-Malus step after claims							
		0	1	2	3	4	5	6	≥ 7
0	250	1	0	0	0	0	0	0	0
1	170	2	0	0	0	0	0	0	0
2	130	3	1	1	0	0	0	0	0
3	110	5	2	2	1	0	0	0	0
4	100	5	3	2	1	0	0	0	0
5	95	6	4	3	2	1	0	0	0
6	90	7	5	4	2	1	0	0	0
7	80	8	6	5	4	2	1	0	0
8	70	8	8	6	5	4	2	1	0

Using the relations (1) and (5) we have:

$$p_{ij}(t) = \{k\} = p_k = \frac{\lambda^k}{k!}e^{-\lambda} \tag{7}$$

or

$$p_{ij}(t) = \{k, k+1, ...\} = \sum_{l=k}^{\infty} p_l \tag{8}$$

For example:

$$p_{53}(t) = \{2\} = p_2 = \frac{\lambda^2}{2!}e^{-\lambda}; \quad p_{50}(t) = \{5,6,7,...\} = \sum_{l=5}^{\infty} p_l.$$

Then, the transition matrix (2) , accordingly to the Romanian model, is:

$$P(t) = \left(p_{ij}(t)\right)_{i,j=\overline{0,8}}.$$

Consequently, the transition matrix could be presented as follows (table no. 2):

Table no. 2

Transition matrix for the Romanian Bonus-Malus system

i/j	0	1	2	3	4	5	6	7	8
0	{1,2,3…}	{0}	.	.	.	.	.	.	.
1	{1,2,3…}	.	{0}	.	.	.	.	.	.
2	{3,4,5…}	{1,2}	.	{0}	.	.	.	.	.
3	{4,5,6…}	.	{1,2}	.	.	{0}	.	.	.
4	{4,5,6…}	{3}	{2}	{1}	.	{0}	.	.	.
5	{5,6,7…}	{4}	{3}	{2}	{1}	.	{0}	.	.
6	{5,6,7…}	{4}	{3}	.	{2}	{1}	.	{0}	.
7	{6,7…}	{5}	{4}	.	{3}	{2}	{1}	.	{0}
8	$\{\geq 7\}$	{6}	{5}	.	{4}	{3}	{2}	.	{0,1}

where . = 0.

Conclusions

The ultimate goal of a Bonus-Malus system is to make everyone pay a premium which is as near as possible the expected value of his yearly claims.

The Bonus-Malus system is the powerful instrument to assess individual risk. Note that for the transitions in the Bonus-Malus system, only the number of claims filed counts, not their size. Although it is clear that a Bonus-Malus system based on claim sizes is possible, such systems are hardly ever used with car insurance.

In conclusion, the basic characteristics of Bonus-Malus system are:

classification according to claims experience (period: 1 year);

number of classes and period required to reach maximum bonus

beginner's class

reclassification at renewal.

As to the reduction of the duality, cooperation between insurers is necessary, in the sense of creating a common database or multilateral agreements, for Bonus-Malus system implementation and the tariffs depending on the risk profile.

By applying the Bonus-Malus system the company assumes the risk of losing many clients disadvantaged by the significant increase of the rate premium, in the next period, but the favorable results will appear on a long or medium term.

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